INVESTIGATION 2

Your task is to investigate the line joining the midpoints of two sides of a triangle. Use of the geometry package on the CD is recommended, but you could also use a ruler and protractor.

**What to do:**

1. Construct a large triangle and label it like the triangle alongside.
2. As accurately as you can, mark the midpoints P and Q of [AB] and [AC] respectively.
3. Join [PQ] and [QB].
4. Measure the lengths of [PQ] and [BC]. What do you notice?
5. Measure $PQ$ and $QB$. What conclusion can be drawn about [PQ] and [BC]?
6. Repeat the procedure with different sized triangles.
7. Copy and complete: “The line joining the midpoints of two sides of a triangle is ...... to the third side and ...... its length.”

From the investigation you should have discovered:

**THE MIDPOINT THEOREM**

The line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

- $[MN]$ is parallel to $[BC]$.
- $MN = \frac{1}{2}(BC)$.

**CONVERSE OF MIDPOINT THEOREM**

The line drawn from the midpoint of one side of a triangle, parallel to a second side, bisects the third side.

$AN = NC$. 
EXERCISE 24F

1. Find the unknowns in the following, giving reasons.

![Diagram a](image1.png)

![Diagram b](image2.png)

2. ABCD is a parallelogram whose diagonals meet at E. M is the midpoint of [AD]. Show that [ME] is parallel to [AB] and half its length.

![Diagram](image3.png)

Example 13

ABCD is a parallelogram. [AB] is produced to E such that AB = BE. [AD] is produced to meet [EC] produced at F. Prove that EC = CF.

![Diagram](image4.png)

In \( \triangle AEF \), [BC] is parallel to [AF].

As ABCD is a parallelogram

Since B is the midpoint of [AE],
C is the midpoint of [EF].

Midpoint theorem converse

Hence EC = CF.

3. ABCD is a triangle. D, E and F are the midpoints of its sides as shown. Show that DFEB is a parallelogram.

![Diagram](image5.png)

4. A and B are the midpoints of sides [PQ] and [PR] of \( \triangle PQR \). Y is any point on [QR]. Prove that X is the midpoint of [PY].

![Diagram](image6.png)
5 Stacey says “I have drawn dozens of different quadrilaterals. When I join the midpoints of adjacent sides of each one of them, the figure formed appears to be a parallelogram.”
   a Draw two quadrilaterals of your own choosing to check Stacey’s conjecture.
   b By drawing one diagonal of a labelled quadrilateral, show that Stacey’s conjecture is correct.

6 To prove the midpoint theorem in \( \triangle BAC \), we extend \([PQ]\) to meet a line from \(C\) which is parallel to \([BA]\) at \(R\). Copy and complete the proof:

   In \( \triangle s APQ \) and \( CRQ \):
   - \( AQ = CQ \) \{.........\} \( ^{(1)} \)
   - \( APQ = CRQ \) \{.........\} \( ^{(2)} \)
   - \( ........ = ......... \) \{vertically opposite\} \( ^{(3)} \)

   \( \therefore \) the triangles are congruent \{.........\} \( ^{(4)} \)

   Consequently, \( AP = CR \), and as \( AP = BP \), \( CR = ....... \) \( ^{(5)} \)

   So, \([BP]\) and \([CR]\) are parallel and equal in length, and this is sufficient to deduce that \(BCRP\) is a \( ......... \) \( ^{(6)} \)

   \( \therefore \) \([PQ]\) is parallel to \( ....... \) \( ^{(7)} \) and \( PR = BC \).

   But, from the congruence, \( PQ = QR \), and so \( PQ = \frac{1}{2} BC \).

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**EULER’S RULE**

Leonhard Euler, pronounced ‘oiler’, was one of the greatest mathematicians of all time. He made numerous interesting observations in geometry. One of them is known as Euler’s Rule. It connects the number of vertices, edges and regions in a polygon.

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**INVESTIGATION 3**

Consider the figure:

- It has 5 vertices,
- 6 edges, and 3 regions.

Outside the figure counts as a region.